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Subject
BOOSTER RF SYSTEM RESPONSE TO GUIDE FIELD 2nd HARMONIC

Guide-Field Equation

In order to study the effect on the Booster-rf system of adding harmonic components to the guide field, a general equation for a guide field with N harmonic components was studied.

$$B(t) = B_{dc} + \sum_{i=1}^{N} B_i \cos(i\omega t + \phi_i).$$

The following constraints were applied:

1) Injection occurs at $t = 0$
   a.) at $t = 0$, $B = B_I$
   b.) at $t = 0$, $\dot{B} = 0$

2) Ejection occurs when $B(t)$ is a maximum
   a.) at $B_{max} = B_E$, $t = t_{max}$
   b.) at t_{max}, $\dot{B} = 0$

Each of the harmonic components may be specified in amplitude and phase relative to the first harmonic, i.e., one may specify R_i, α_i; $2 \leq i \leq N$, where $R_i = B_i/B_1$, $\alpha_i = \phi_i - \phi_1$. $(R_1 \equiv 1, \alpha_1 \equiv 0)$

After stating the above conditions, one is left with the task of determining B_{dc}, B_1, ϕ_1, and t_{max}.

At injection, $t = 0$ and $\dot{B} = 0$.

$$\dot{B}(t) = -\omega \sum_{i=1}^{N} iB_i \sin(i\omega t + \phi_i)$$

$$\dot{B}\Big|_{t=0} = 0 = \sum_{i=1}^{N} iB_i \sin \phi_i$$

Since $B_i = R_i B_1$ and $\phi_i = \phi_1 + \alpha_i$,

$$\sum_{i=1}^{N} iR_i \sin(\phi_1 + \alpha_i) = 0.$$

R_i and ϕ_i have been specified, so this equation determines the parameter ϕ_1, and hence ϕ_i, $2 \leq i \leq N$.

At ejection, $t = t_{max}$ and $\dot{B} = 0$.

$$\dot{B}\Big|_{t=t_{max}} = 0 = \sum_{i=1}^{N} iB_i \sin(i\omega t_{max} + \phi_i)$$

$$\sum_{i=1}^{N} iR_i \sin(i\omega t_{max} + \phi_i) = 0$$

Since R_i was specified and ϕ_i determined, this equation determines t_{max}.

At injection, $t = 0$ and $B = B_I$.

$$B_I = B_{dc} + \sum_{i=1}^{N} R_i B_1 \cos\phi_i$$

At ejection, $t = t_{max}$ and $B = B_E$.

$$B_E = B_{dc} + \sum_{i=1}^{N} R_i B_1 \cos(i\omega t_{max} + \phi_i)$$

Combining these two equations to eliminate B_{dc}, one obtains

$$\sum_{i=1}^{N} R_i B_1 [\cos(i\omega t_{max} + \phi_i) - \cos\phi_i] - (B_E - B_I) = 0$$

which determines B_1 and hence B_i. Then,

$$B_{dc} = B_I - \sum_{i=1}^{N} B_i \cos\phi_i .$$

Since the equations for ϕ_1, t_{max}, and B_1 are not explicit, they are solved using a successive approximation technique.

RF-System Response

The effect of adding second harmonic with a variety of amplitudes and phases was studied with the aid of the RFCOST computer program.

Beneficial effects of an added 2nd harmonic in the guide field:

1) Reduction of maximum $\dot{B}$ and hence, reduced maximum voltage/turn requirement.

2) Lengthened guide-field rise time and shortened fall time, improving the duty factor of acceleration.

3) Reduction (slight) of the maximum phase oscillation frequency.

Numbers 1) and 2) combined permit acceleration with fewer cavities. Number 3) is in a direction helpful to beam control; the peak phase-oscillation frequency is reduced by 5% (ν_s max without 2nd harmonic is .083, with 2nd harmonic, .079).

For the present booster and rf-system parameters, a cost minimum exists with

$$B_2/B_1 = -1/8, \ \phi_2 - \phi_1 = -1.$$

This results in the following guide-field equation parameters:

$B_{dc} = 4036$ Gauss, $B_1 = -3812$ Gauss, $B_2 = 477$ Gauss,

$\phi_1 = -.239$, $\phi_2 = -1.239$.

As shown in Figure 1, acceleration time is lengthened, and $\dot{B}$ is reduced, particularly early in the acceleration time when ferrite losses dominate the rf system. The effect on total power for one accelerating module is evident in Figure 2.

As one might expect, the cost minimum occurs when the two power peaks are equal as in the 2nd harmonic case shown.

The case described results in a possible reduction of the number of booster-rf cavities from 16 to 14 with a cost savings of about $500 K. It should be noted that changes in booster or rf-system parameters are likely to move the location of the cost minimum, so that any plans to take advantage of the promised savings should include facilities for adjusting the relative amplitude and phase of the second-harmonic component.

The addition of harmonics to the guide field by means of pole-tip saturation is a matter not considered in this note. If the optimum 2nd-harmonic guide field is modified by the superposition of pole-tip saturation, the effect may be to increase the rf-system cost because the phase and amplitude of the resulting harmonics are no longer optimum.

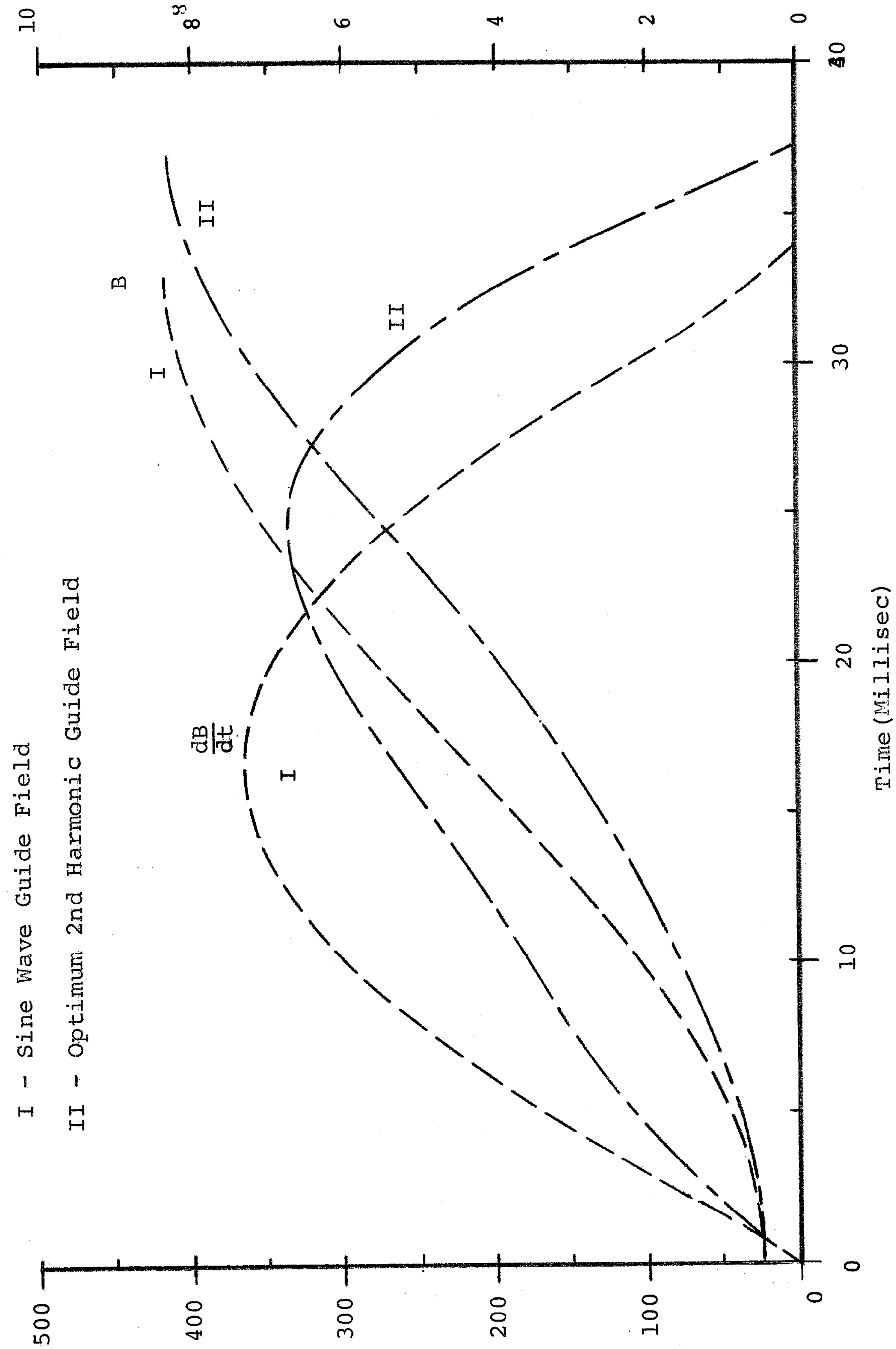

Figure 1. B and dB/dt vs. Time

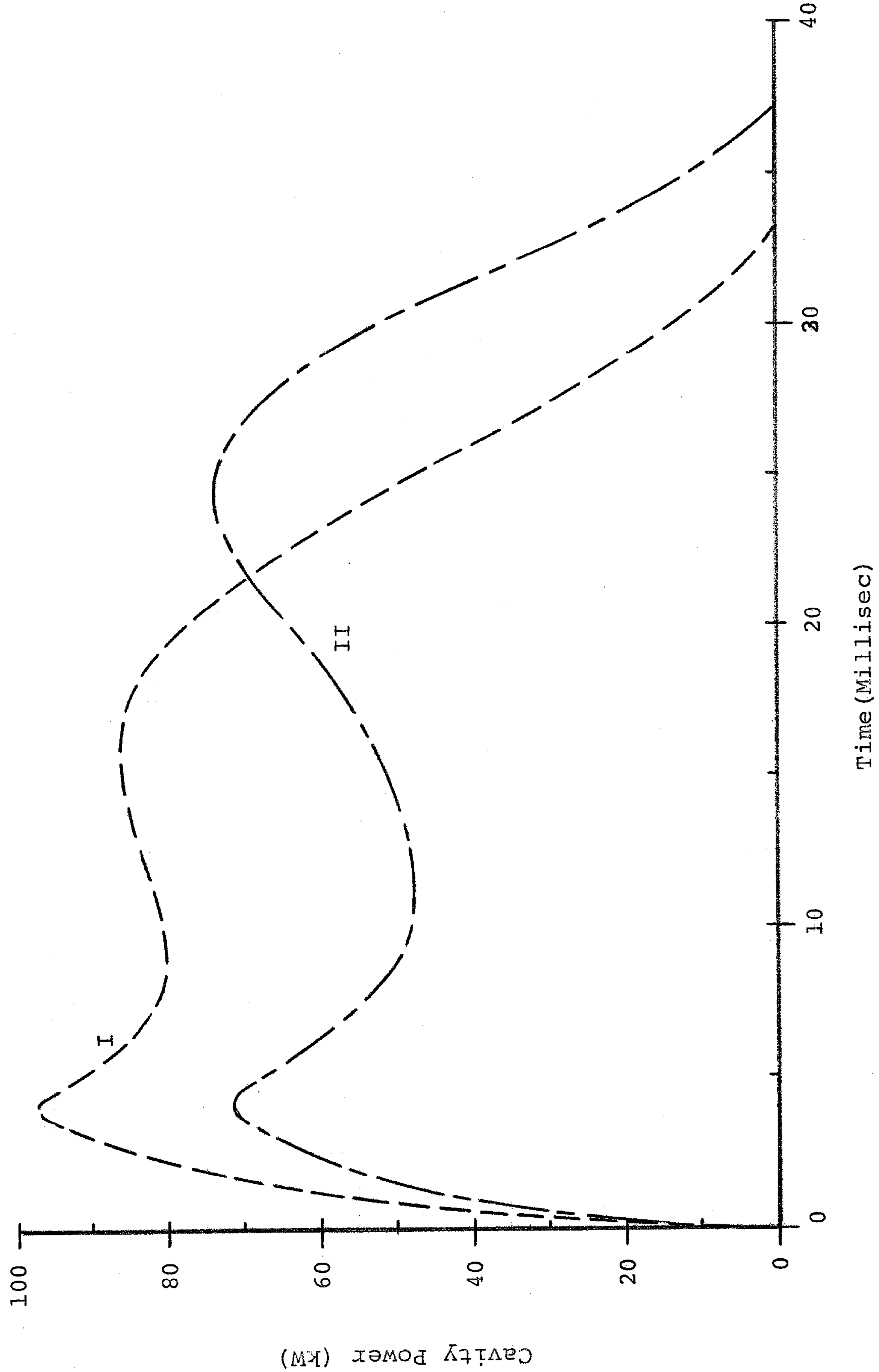

Figure 2. Cavity Power vs. Time